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CORRECTIONS APPLIED BY THE NATIONAL GEODETIC SURVEY TO PRECISE LEVELING OBSERVATIONS

Rockville, Md.
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ABSTRACT. The six corrections applied by the National Geodetic Survey to precise leveling observatons are described. These corrections are applied to observed leveling data to minimize the effects of known systematic errors. The rod scale correction ensures a uniform scale which conforms to the National length standard. The rod temperature correction accounts for variation in the length of the leveling rod's Invar/LO-VAR® strip which results from temperature changes. The level collimation correction minimizes the error caused by nonhorizontality of the leveling instrument's line of sight for unequal sight lengths. The refraction correction is modeled to minimize the refraction error caused by temperature (density) variation of air strata. The astronomic correction counteracts the effects of the Moon and Sun on the equipotential surfaces of the Earth. The orthometric correction eliminates the effect of the nonparallelism of equipotential surfaces. The height system presently used at the National Geodetic Survey and those height systems proposed for the future are also described.

INTRODUCTION

To achieve the highest degree of accuracy in the measurement of elevation differences, the National Geodetic Survey (NGS), a main line component of the National Ocean Survey, formerly the Coast and Geodetic Survey (C&GS), uses the most modern leveling equipment available. Observational procedures have also been designed to provide the most effective method to acquire data. In addition, those systematic errors which cannot be sufficiently controlled by instrumentation or observational techniques are minimized by applying appropriate corrections to the observed data.

Rod scale and rod temperature corrections have been applied to observed height differences since the beginning of precise leveling by C&GS in 1878. The level collimation correction has been applied since about 1900, when the Fischer leveling instrument was introduced. Orthometric corrections have been applied to observed elevation differences since 1910, using the formula later adopted by the Seventeenth General Conference of the International Association

of Geodesy in 1912 (Bowie and Avers 1914). Refraction and astronomic corrections have been applied to leveling data since 1973.

ROD SCALE CORRECTION BASED ON THE CALIBRATION OF SEVERAL ROD GRADUATIONS

The length of the Invar strip of a leveling rod used in precise leveling should be traceable to the National length standard. Precise leveling rods should be calibrated before and after each project if practical, whenever possible damage has occurred, or at least once during each year of use. The calibrated length of a rod is usually determined by comparing its Invar strip to a standard meter. The length excess of an average rod meter is computed from the "actual minus nominal" length differences observed at several points along the rod. A rod scale correction is applied to the observed difference of elevation between permanently monumented vertical control points (bench marks) using an average length excess for the rod pair. The accuracy of the rod scale correction depends on the accuracy of the rod length excess determination and the stability of the Invar scale with time. The correction is computed by the following formula (Rappleye 1948) and added with the resultant algebraic sign to the observed elevation difference:

$$C_r = De \qquad (1)$$

where C_r = rod scale correction in millimeters,

D = observed difference of elevation for the section in meters, and

e = average length excess of the rod pair in mm/m.

Example: Standardization reports prepared by the National Bureau of Standards for two leveling rods contain the following information:

1. Standardization temperature: 25°C.
2. Coefficient of thermal expansion: 0.0000008 per meter per degree Celsius.
3. Actual length between designated points of the rods at the standardization temperature:

Nominal length:	0-40	0-200	0-400	0-600	in half-centimeters
Rod 1	0.19990	0.99982	1.99985	2.99982	in meters
Rod 2	0.19994	0.99998	1.99994	2.99994	in meters.

The computed length excesses of an average rod meter are: Rod 1: -0.0213 mm/m; Rod 2: -0.0027 mm/m; and for the rod pair: -0.0120 mm/m.

The computed index errors are: Rod 1: -0.119 mm; Rod 2: -0.046 mm.

The length excess and index error are determined using a least-squares adjustment. The index error is the difference between the "imaginary" zero mark of the scale and the reference point on the bottom of the rod frame. The effect of index errors is minimized by appropriate field procedures; e.g., "leap-

frogging" the leveling rods and using an even number of setups between bench marks, or "matching" rod pairs with (nearly) equal index errors.

ROD SCALE CORRECTION BASED ON THE CALIBRATION OF ALL ROD GRADUATIONS

In 1980 the National Bureau of Standards developed, as specified in a NGS contract, an automated measurement system for the "detailed" calibration of geodetic leveling rods (Balazs 1980). The system incorporates a 6-meter, one-dimensional measuring machine, a motorized carriage, a photoelectric microscope, and a helium-neon laser interferometer interfaced to a minicomputer, to determine graduation locations on linear scales. Measurements can be made of virtually any leveling rod scale with scribed, engraved, or painted graduations. Data acquisition is triggered by light-intensity variations sensed by a photodetector, which allows both edges of every graduation on the Invar strip of a leveling rod to be measured. The laser interferometer provides a length measurement referenced to either a fixture mounted to the foot piece of the rod or to any graduation on the rod scale itself. The accuracy (standard error) of the system is reported to be better than ± 50 micrometers. (A comprehensive error analysis has not yet been completed.)

When detailed calibration data became available in late 1980, a new procedure for applying the rod scale correction was implemented at NGS for those leveling rods which had received the detailed calibration. The procedure determines the standard length between the center of the rod foot plate, in line with the scale and each line graduation. Standard meter distances are stored in a computer file along with the nominal rod readings for the corresponding line graduations. The calibration value for each rod reading is obtained by entering the file with the nominal rod reading (without the micrometer reading) and selecting the corresponding standard meter value. The micrometer reading is converted to meters and added to the standard meter value to obtain the height in meters above the rod foot plate. The index correction (explained previously) is not needed when the detailed rod calibration is used. The height above the rod foot plate must be corrected for the difference between survey-and-calibration Invar temperatures using the coefficent of thermal expansion explained next in the "Rod Temperature Correction" section.

ROD TEMPERATURE CORRECTION

The coefficent of thermal expansion of the Invar strip of a NGS leveling rod is determined by the National Bureau of Standards. The calibrated length of an average rod meter is given for each rod for a specific temperature, as determined from the rod excess computation and standardization. The rod temperature correction is applied to the elevation difference between bench marks using the mean of the Invar temperatures observed at the beginning and end of a section. (A section is the interval between adjacent permanently monumented bench marks.) The correction is computed by the following formula (Rappleye 1948) and added

with the resultant algebraic sign to the observed elevation difference for the section:

$$C_t = (t_m - t_s)\ DCE \qquad (2)$$

where C_t = rod temperature correction,

t_m = mean observed temperature of the Invar strip,

t_s = standardization temperature of the Invar strip,

D = observed difference of elevation between the bench marks, and

CE = mean coefficient of thermal expansion per unit length per degree temperature of the rod pair.

The units of D, t_m, and t_s must be consistent with the unit of CE. The unit of C_t is the same as the unit of D.

LEVEL COLLIMATION CORRECTION

The effects of the collimation error of a leveling instrument are best minimized by field procedures. If sight lengths are balanced, i.e., DS = 0 and SDS = 0, where DS is the difference between backward and forward sight lengths at one setup (DS = backsight distance - foresight distance) and SDS is the accumulated DS for a section, the effect of the collimation error approaches zero. A well-adjusted instrument also minimizes this error without balancing sight lengths, although the collimation error of most leveling instruments changes slightly throughout the day as a result of changing temperature.

When SDS or the collimation error of the instrument is not zero, a correction is added with the resultant algebraic sign to the observed difference of elevation of each running of a section using the following formula (modified from Rappleye 1948):

$$C_c = - (e\ SDS) \qquad (3)$$

where C_c = level collimation correction in millimeters,

e = collimation error in radians x 1,000 or in mm/m, and

SDS = accumulated difference in sight lengths for the section in meters.

For the Zeiss Ni 002 leveling instrument, the collimation error should be determined at least weekly during the period when the instrument is in use. For

most other instruments, the collimation error should be determined each day during use. Several methods have been developed to determine level collimation error (Jordan et al. 1956, Rappleye 1948). Level collimation error must not be determined when the temperature gradient at the observing site is positive. A positive temperature gradient occurs when the air nearer the ground is colder than the air directly above the same point. Additional details about measuring temperature differences are stated in the "Refraction Correction" section. If equipment is not available to determine the temperature gradient, the collimation error should be determined during the time of day between 2 hours after sunrise and 2 hours before sunset. If possible, the surveying team should avoid frozen or snow-covered ground when determining the collimation error.

REFRACTION CORRECTION

A simplified version of the model developed by Professor T. J. Kukkamaki of the Finnish Geodetic Institute (Kukkamaki 1939) is used to compute corrections to observed elevation differences of a section to minimize the effect of refraction:

$$R = - 10^{-5} \gamma \{s/[(2n)(50)]\}^2 \delta dW \qquad (4)$$

where R = refraction correction in millimeters for the section,
γ = 70 (Hytonen 1967),
s = section length in meters,
n = number of setups,
δ = "predicted" temperature difference in degrees Celsius between temperatures at 2.5 m and 0.5 m above the ground (upper temperature minus lower temperature),
d = difference of elevation for the section in units of half-cm, and
W = weather factor (based on the recorded "sun code"). The weather factor W is as follows: 0.5 for 100 percent cloudy sky, 1 for 50 percent sunny sky, 1.5 for 100 percent sunny sky.

Note: W is applied only when δ is predicted as explained in the next two paragraphs. For observed δ, use the original Kukkamaki model shown in eq. 5.

From 1973 to 1980, NGS used the temperature differences tabulated by Best (1935) for 0.3 m and 1.2 m above the ground as the predicted values for δ. Refraction experiments conducted by NGS in 1980 (Whalen 1980) indicated that the temperature differences tabulated in the Best tables (Best 1935) were often too small for the United States.

Holdahl (1981) has developed a model to predict temperature gradients for the conterminous United States from historic meteorological data, for use when observed temperature differences are not available. Refraction corrections for NGS leveling data that do not include observed temperature differences are now being recomputed with new values predicted by Holdahl's model (Holdahl 1981).

In the fall 1980, NGS leveling units began observing two temperatures at each instrument setup to determine δ. When these two values are observed at each setup, the refraction correction is computed with the original Kukkamaki model (Kukkamaki 1939) and added with the resultant algebraic sign to the observed elevation difference at a setup:

$$R = - 10^{-5} \gamma (S/50)^2 \delta D \qquad (5)$$

where S = sight length (instrument to rod) in meters,
δ = observed temperature difference in degrees Celsius between temperatures observed at 1.3 and 0.3 m (upper temperature minus lower temperature) at each setup, and
D = difference of elevation for the setup in units of half-cm.
γ is as defined previously.

It is assumed that δ observed for the 1.3- and 0.3-meter interval is not significantly different from δ observed for the 2.5- to 0.5-meter interval recommended by Kukkamaki.

ASTRONOMIC CORRECTION

The astronomic correction is applied to account for the effect of tidal accelerations due to the Moon and Sun on the Earth's equipotential surfaces. The astronomic correction is small, amounting, at most, to 0.1 mm/km, but it accumulates in the north-south direction. For example, the maximum accumulated astronomic correction between Spokane, Wash., and San Diego, Calif., is 7 cm. From Alaska to Panama, it accumulates to 15 cm (Holdahl 1979). It is not usually necessary to apply the astronomic correction to local leveling surveys, but it should be applied for systems of regional or continental extent.

In the past, the astronomic correction was difficult to compute because it required the zenith distances and azimuths of the Sun and Moon. Today, computer subroutines easily calculate these quantities. The required input for the correction is: time and date of measurements, heights of the bench marks, and geodetic positions of the "From" and "To" bench marks. It is applied separately to forward and backward runnings over the same line of leveling since each running is observed at a different time.

The basic formulae to compute the vertical and horizontal components of tidal acceleration, g_o and h_o, respectively, on a rigid Earth have been published by several authors, e.g., Schureman (1924), Pettit (1954), Jordan et al. (1956), and Longman (1959). NGS uses the equations developed and published by Longman (1959) to compute the horizontal components of tidal acceleration (only the horizontal components influence elevation differences):

$$h_m = (3\mu Mr)(\sin 2\theta)/2d^3 + (3\mu Mr^2)(5 \cos^2 \theta - 1)(\sin \theta)/2d^4 \qquad (6)$$

$$h_s = (3\mu\ Sr)(\sin 2\rho)/2D^3 \qquad (7)$$

where h_m = horizontal component of tidal acceleration due to the Moon,

h_s = horizontal component of tidal acceleration due to the Sun,

μ = Newton's gravitational constant,

r = distance from the point on the surface to the center of the Earth,

M = mass of the Moon,

S = mass of the Sun,

d = distance between the centers of the Earth and Moon,

D = distance between the centers of the Earth and Sun,

θ = zenith distance of the Moon, and

ρ = zenith distance of the Sun.

Additional equations and constants are given in Longman (1959). The equation to compute the inclination of the Earth's equator to the ecliptic is not given in Longman (1959). It was taken from the American Ephemeris and Nautical Almanac (U.S. Naval Observatory):

$$W = 23.4522944 - 0.0130125\ T - 0.164 \times 10^{-5}\ T^2 + 0.503 \times 10^{-6} T^3 \qquad (8)$$

where W = inclination of the Earth's equator to the ecliptic in degrees, and
T = number of Julian centuries.

There are 36,525.0 days in a Julian century. The days are counted from December 31, 1899, Greenwich mean noon, to the time of observation.

After h_m and h_s are computed, they are converted to deflections (ε) by dividing them by a gravity value (Simonsen 1965). A constant gravity value of 980.39 mgal is used:

$$\varepsilon_m = h_m/980.39 \qquad (9)$$

$$\varepsilon_s = h_s/980.39 \qquad (10)$$

where ε_m = deflection due to the Moon, and

ε_s = deflection due to the Sun.

The azimuth and distance of the section are determined from the positions of the "beginning" and "ending" bench marks.

The correction (C_a^l) to minimize the effect of the Sun and Moon is computed by the following equation (Jensen 1949):

$$C_a^l = Ks \tag{11}$$

where C_a^l = astronomic correction on a rigid Earth,

s = section length,

$K = \tan \varepsilon_m \cos (A_m - \alpha) + \tan \varepsilon_s \cos (A_s - \alpha)$,

A_m = azimuth of the Moon,

A_s = azimuth of the Sun, and

α = azimuth of the section.

Because the Earth's crust is elastic, this correction is reduced by 30 percent in NGS applications (Vanicek 1980). The final form (which is applied to the observed elevation difference of the section) is:

$$C_a = 0.7 \cdot Ks \tag{12}$$

where C_a = astronomic correction for an elastic Earth, and

K and s are as defined previously.

ORTHOMETRIC CORRECTION

The following formula is used to compute the approximate orthometric correction (based on normal gravity) to the observed difference in elevation of a section. The theory and derivation of the formula are published in Special Publication 240 (Rappleye 1948: appendix C), and in Vanicek et al. (1980):

$$C_o = -2h\alpha \sin 2\rho \, [1 + (\alpha - 2\beta/\alpha)(\cos 2\rho)] \, d\rho \tag{13}$$

where C_o = orthometric correction,

h = average height of the section,

$\alpha = 0.002644$,

β = 0.000007,

ρ = average latitude of the section, and

$d\rho$ = latitude difference between the "beginning" and "end" points of the section. ($d\rho$ is positive when the "end" point is north of the "beginning" point).

The unit of orthometric correction is the same as the unit of h. The orthometric correction is applied to the observed elevation difference of the section (elevation of the "end" point minus the elevation of the "beginning" point). The orthometric correction is not needed when geopotential numbers (explained in the next section) are used.

HEIGHT SYSTEMS

By applying the orthometric correction and the other five corrections explained in this publication, NGS obtains a "best estimate" of observed, normal orthometric elevation differences, i.e., observed orthometric elevations based on normal (not observed) gravity values. Gravity anomalies along leveling routes have not been incorporated into height differences in the past since, until the recent advent of modern gravimeters, it was not practical to measure gravity on a routine basis along leveling lines.

Observed, normal orthometric elevation differences have been used historically by NGS as input (observables) to least-squares adjustments to determine adjusted ("final") normal orthometric elevation differences. By incorporating appropriate normal orthometric heights of previously adjusted bench marks during the adjustment process, consistent adjusted normal orthometric heights, referred to a particular vertical reference surface (e.g., the National Geodetic Vertical Datum of 1929), are obtained. Adjusted normal orthometric heights referenced to the National Geodetic Vertical Datum of 1929 (NGVD 29) are the heights presently published and distributed by NGS.

The expressions for approximate orthometric corrections (based on normal gravity) were developed by Bowie and Avers (1914) and are still used today in North America. NGS-published elevations referred to NGVD 29 are based on this system. Normal gravity for the approximate orthometric correction is computed with the equivalent of Helmert's International formula (Rappleye 1948):

$$g' = \gamma_{45}(1 - \alpha \cos 2\rho + \beta \cos^2 2\rho - kh) \qquad (14)$$

where g' = normal gravity at latitude ρ and orthometric elevation h,
γ_{45} = 980.624 gals: the normal acceleration of gravity at sea level at a latitude of 45°,

α = 0.002644,
β = 0.000007, and
k = 0.0000003147, if h is in meters.

The relationship between dynamic height (H) and potential (W) was used to derive the formula used to compute the orthometric correction (Rappleye 1948). The dynamic height of a point is derived from its geopotential number. The geopotential number is the difference between the potential of the geoid and the potential of the surface on which the point resides, and is expressed by the following formula (Heiskanen and Moritz 1967):

$$C = \int_{O}^{A} g\,dn \qquad (15)$$

where C = geopotential number,
O = a particular point on the geoid,
A = another point, connected to O by a line of precise leveling,
g = average value of actual gravity between the successive bench marks, and
dn = elevation difference of the successive bench marks.

Geopotential number is measured in geopotential units (gpu), where 1 gpu = 1 kgal meter = 1,000 gal meters (Heiskanen and Moritz 1967). The use of geopotential numbers (instead of dynamic heights) was adopted by the International Association of Geodesy in 1955. Dynamic height differs from geopotential number only in scale, since division by a constant merely converts the geopotential number to a length.

The use of geopotential numbers (instead of orthometric heights based on normal gravity) to represent the points of the National Geodetic Vertical Control Network is very advantageous. For example, geopotential differences between two points are independent of the route along which the leveling is run; geopotential differences also provide an accurate measure of the "hydrostatic head" between points, i.e., if mean water surfaces of oceans or lakes represent a particular level surface, that surface has the same geopotential number at all locations. In the Great Lakes area of Canada and the United States, another vertical control system is in use by the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data. This system, the International Great Lakes Datum of 1955, uses dynamic heights.

"True" orthometric heights are based on <u>actual</u> gravity values. In the next general readjustment of the National Geodetic Vertical Control Network, now underway at NGS, geopotential numbers, reflecting actual gravity, will be computed for all bench marks. Geopotential numbers will be converted to true orthometric heights after the new adjustment of the North American Vertical Datum (NAVD 88). Unlike orthometric heights based on normal gravity, these orthometric heights will reflect <u>actual</u> gravity at all points. The most appropriate formula to convert geopotential numbers to true orthometric heights for NAVD 88 purposes has not yet been selected.

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