Multi-GNSS Single-Difference Baseline Processing at NGS with newly developed M-PAGES software

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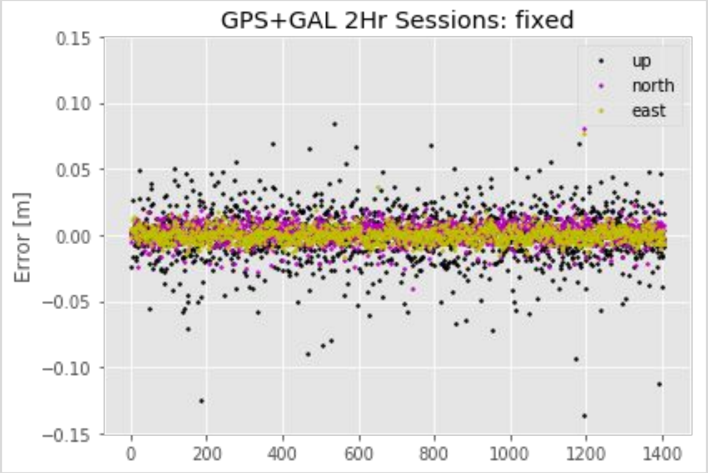
- **M-PAGES** = **M**ulti-GNSS **PAGES** Software
- Single-difference baseline positioning model
- Software will integrated for use in:
  - Online Positioning User Service (OPUS)
  - GNSS Orbit determination
  - NOAA CORS Network (NCN) monitoring

Systems	Solution Type	East RMS [cm]	North RMS [cm]	Up RMS [cm]
GPS-only	Float	2.52	1.22	3.43
	Fixed	1.07	0.88	2.53
GPS+GAL	Float	1.32	0.97	2.60
	Fixed	0.57	0.76	2.24



Above: East/North/Up positioning errors for GPS+GAL fixed solutions.

Left: Positioning results for ~30 baselines (< 200 km; 45 x 2-Hr sessions each).

Overview

- M-PAGES = **M**ulti-GNSS **PAGES** Software
- Single-difference baseline processing strategy
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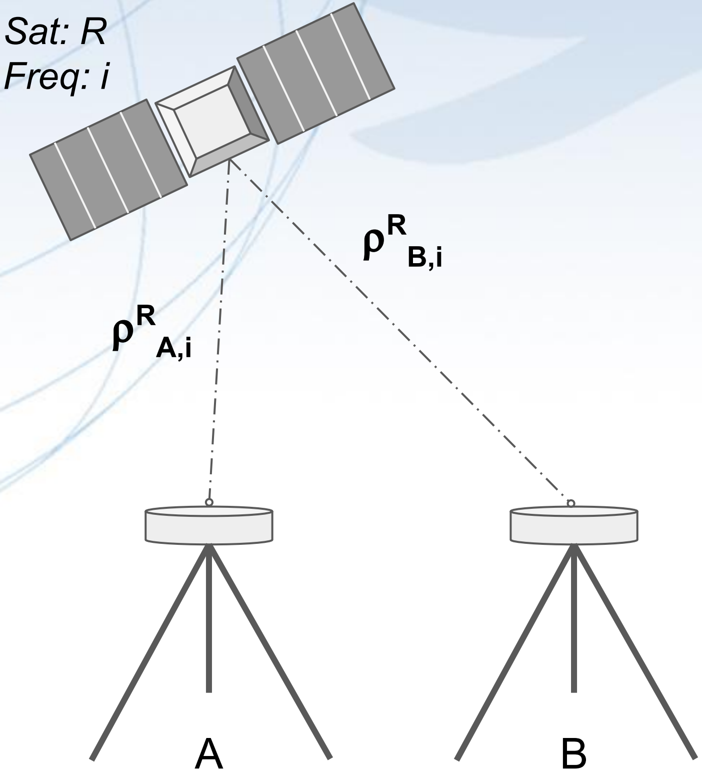
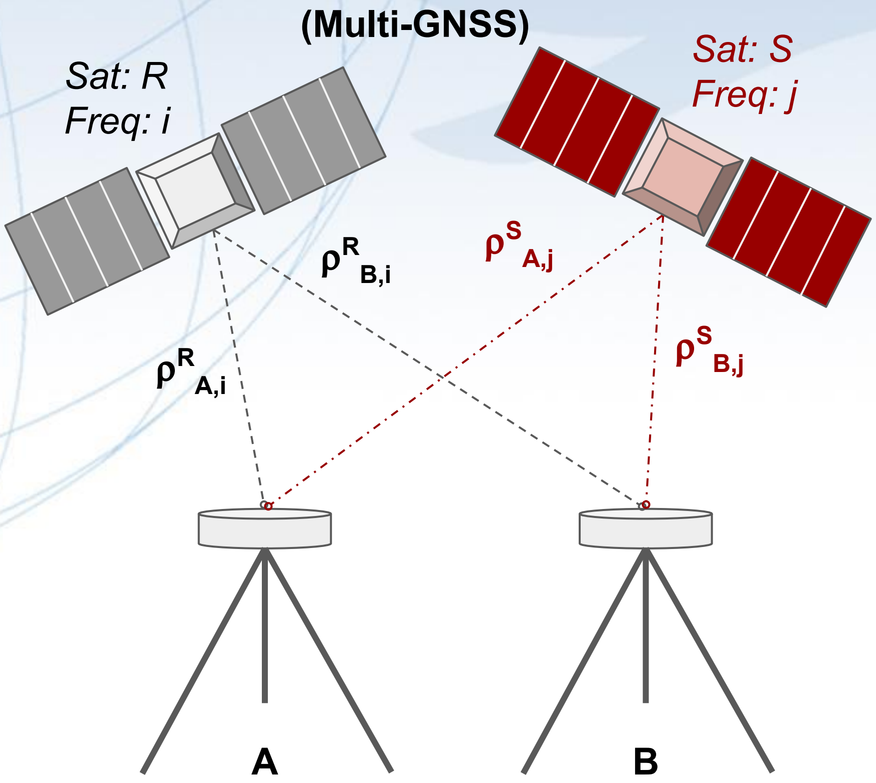
Single-Difference Baseline Processing



SD: $\rho^R_{AB,i} = \rho^R_{A,i} - \rho^R_{B,i}$

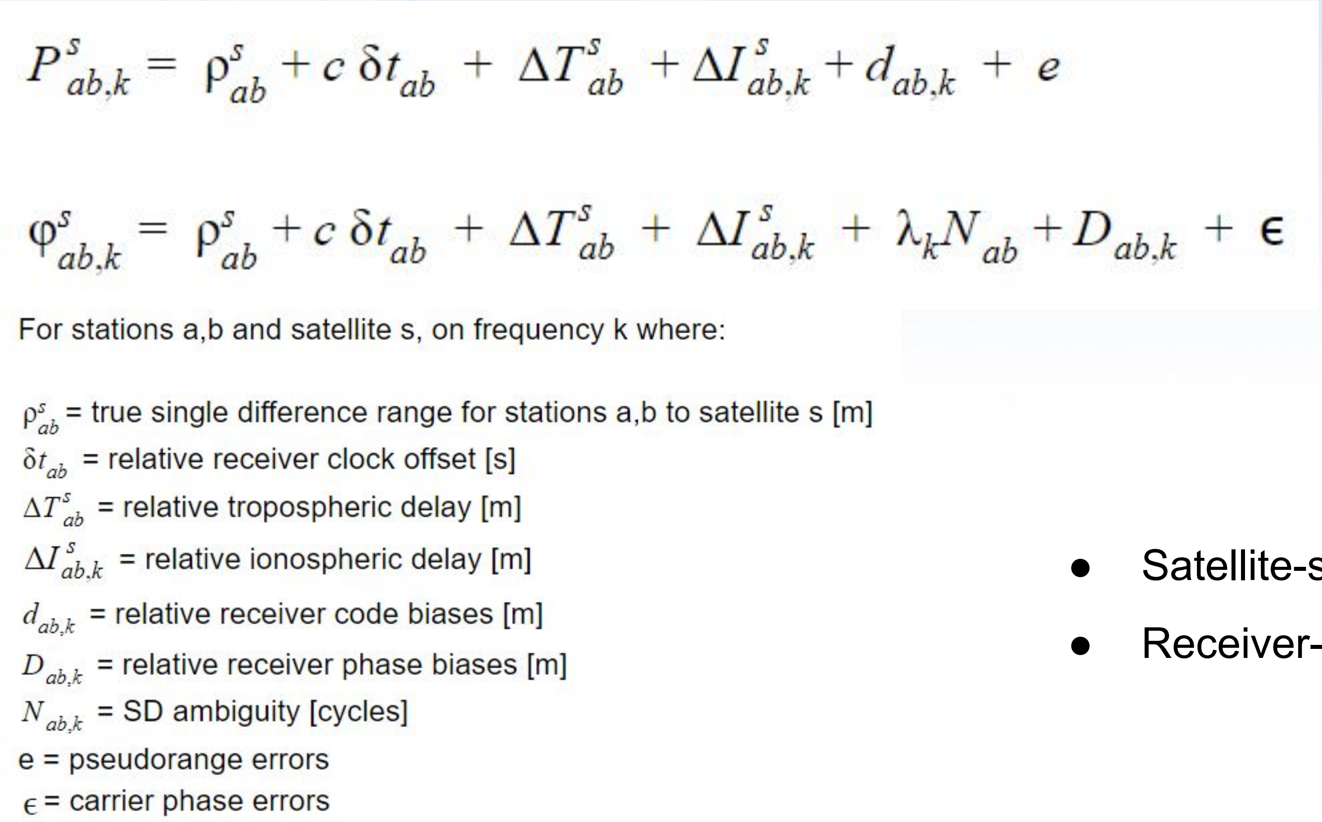
- Satellite-specific terms cancel
- Receiver terms (clock, biases) do not
- Flexible for multi-GNSS
    - All SD observables are on the same frequency

Why not double-difference?



- If frequency i ≠ j we cannot resolve integer ambiguities
- Tightly coupled double-difference processing is possible (e.g., GPS L1/L5 and GAL E1/E5a)
  - Limits processing to GPS satellites with L5
- Could process separately then combine normal equations
- Overall, we prefer the single-difference model for multi-GNSS

Single-Difference Equations



$$P^s_{ab,k} = \rho^s_{ab} + c\,\delta t_{ab} + \Delta T^s_{ab} + \Delta I^s_{ab,k} + d_{ab,k} + e$$

Pseudorange

$$\varphi^s_{ab,k} = \rho^s_{ab} + c\,\delta t_{ab} + \Delta T^s_{ab} + \Delta I^s_{ab,k} + \lambda_k N_{ab} + D_{ab,k} + \epsilon$$

Carrier Phase

For stations a,b and satellite s, on frequency k where:

ρ^s_{ab} = true single difference range for stations a,b to satellite s [m]
δt_{ab} = relative receiver clock offset [s]
ΔT^s_{ab} = relative tropospheric delay [m]
$\Delta I^s_{ab,k}$ = relative ionospheric delay [m]
$d_{ab,k}$ = relative receiver code biases [m]
$D_{ab,k}$ = relative receiver phase biases [m]
$N_{ab,k}$ = SD ambiguity [cycles]
e = pseudorange errors
ϵ = carrier phase errors

- Satellite-specific terms drop out
- Receiver-specific terms do not

Single-Difference Ionosphere-Free Equations

$$P^{s}_{ab,\,if} = \frac{f_i^2 P^{s}_{ab,\,i} - f_j^2 P^{s}_{ab,\,j}}{f_i^2 - f_j^2} = \rho^{s}_{ab} + c\,\delta t_{ab} + \Delta T^{s}_{ab} + d^{s}_{ab,if}$$

Pseudorange

$$\varphi^{s}_{ab,\,if} = \frac{f_i^2 \varphi^{s}_{ab,i} - f_j^2 \varphi^{s}_{ab,\,j}}{f_i^2 - f_j^2} = \rho^{s}_{ab} + c\,\delta t_{ab} + \Delta T^{s}_{ab} + \lambda_{if} N^{s}_{ab} + D^{s}_{ab,if}$$

Carrier Phase

- When processing multiple systems an inter-system bias terms must be introduced.
- Code bias term (d_{ab}^{s}) drops out for reference combination (i.e., GPS C1W/C2W)
- To make ambiguity resolution possible, we first solve for the wide-lane ambiguity using either the wide-lane or Melbourne-Wübbena combination:

$$\varphi^{s}_{ab,\,if} = \frac{f_i^2 \varphi^{s}_{ab,i} - f_j^2 (\varphi^{s}_{ab,\,j} + \lambda_j N_{wl})}{f_i^2 - f_j^2} = \rho^{s}_{ab} + c\,\delta t_{ab} + \Delta T^{s}_{ab} + \lambda_{nl} N^{s}_{ab,nl} + D_{ab,if}$$

λ_{nl} = narrow lane wavelength (~10.6 cm for GPS L1/L2)

Single-Difference Wide-Lane Equations

$$P_{ab,wl} = \frac{f_i P_{ab,i} - f_j P_{ab,j}}{f_i - f_j} = \rho^s_{ab} + c\,\delta t_{ab} + \Delta T^s_{ab} + \Delta I^s_{ab,wl} + d_{ab,wl}$$

Pseudorange

$$\varphi_{ab,wl} = \frac{f_i \varphi_{ab,i} - f_j \varphi_{ab,j}}{f_i - f_j} = \rho^s_{ab} + c\,\delta t_{ab} + \Delta T^s_{ab} + \Delta I^s_{ab,wl} + D_{ab,wl} + \lambda_{wl} N^s_{ab,wl}$$

Carrier Phase

- On **short baselines**, we can neglect the effects of the troposphere and ionosphere and still resolve the wide-lane ambiguities.
    - Residual effects are small relative to the wide-lane wavelength (~86 cm for GPS L1/L2)
- On **long baselines**, we must either model/estimate these effects or use the geometry-free Melbourne-Wübbena combination to estimate wide-lane ambiguities.

Single-Difference Melbourne-Wübbena Equations

$$MW = \varphi_{wl} - P_{nl}$$

$$MW = \lambda_{wl} N_{wl} + D_{ab,\,wl} + d_{ab,nl}$$

- Combination is geometry-free and therefore should not be impacted by baseline length
- Higher noise level due to usage of pseudorange

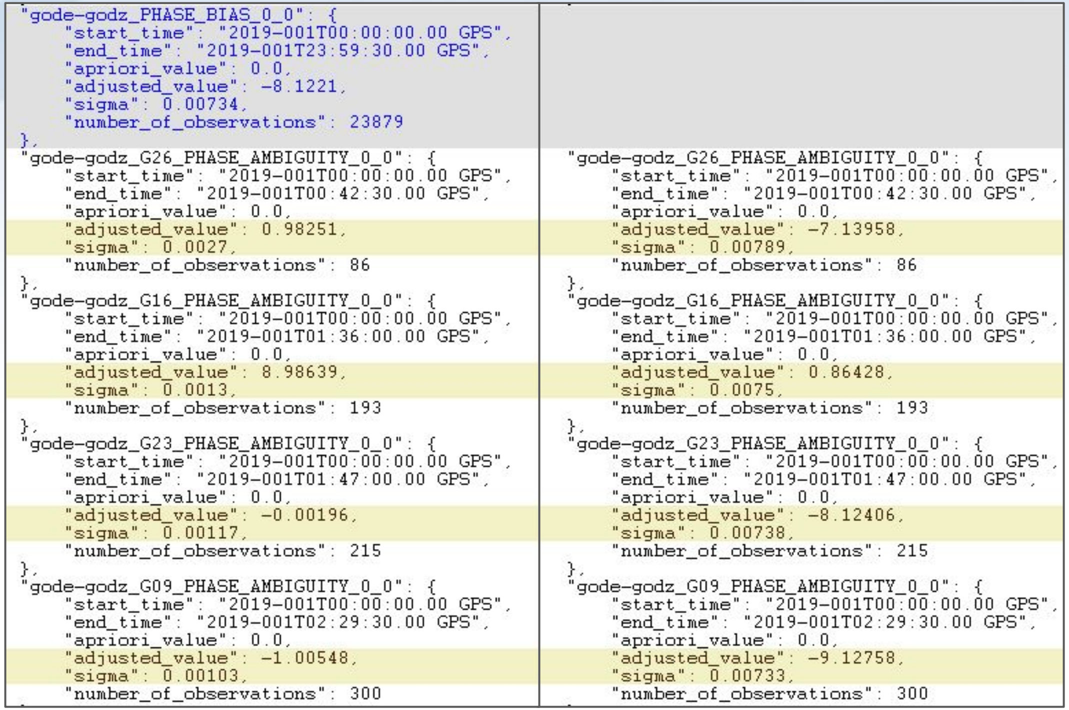
- Combining the code and phase bias terms, assuming they remain stable:

$$MW = \lambda_{wl} N_{wl} + B_{ab,mw}$$

- This yields a biased estimate of the single-difference wide-lane ambiguity.

Ambiguity Resolution

- Receiver phase bias terms do not cancel which yields biased float ambiguity estimates.
- To combat this, we select a “datum” arc (per frequency) to absorb the effect.
  - Datum arc: contribute to phase bias parameter
  - All other arcs: contribute to phase bias and ambiguity parameters
- As a result, we are able to resolve the single-difference ambiguities.
- M-LAMBDA approach



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```

Float ambiguity estimates are very close to integers when phase bias term is introduced (left). Without phase bias, float ambiguities exhibit a consistent decimal component (right).

Sample Results- GPS Only

- ~30 baselines (< 200 km) selected from the NOAA CORS Network
- 45 x 2-Hr sessions (2021-001 - 2021-045)
- Single-baseline solutions evaluated against ITRF2014 station coordinate functions

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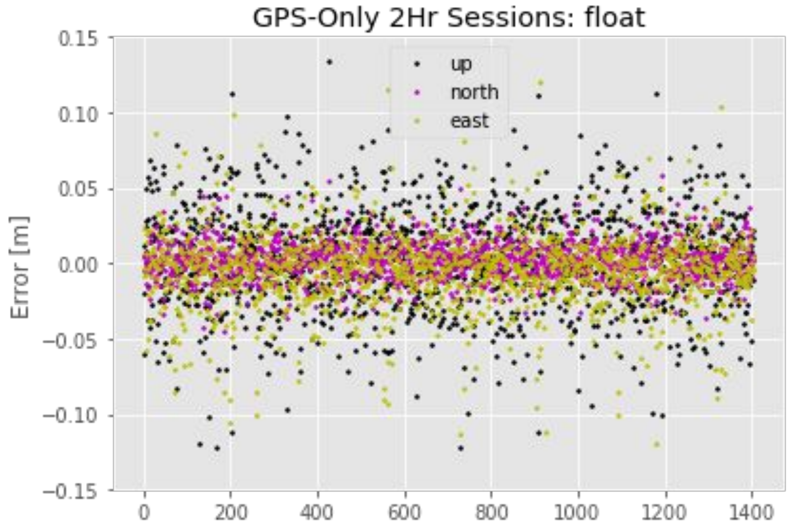

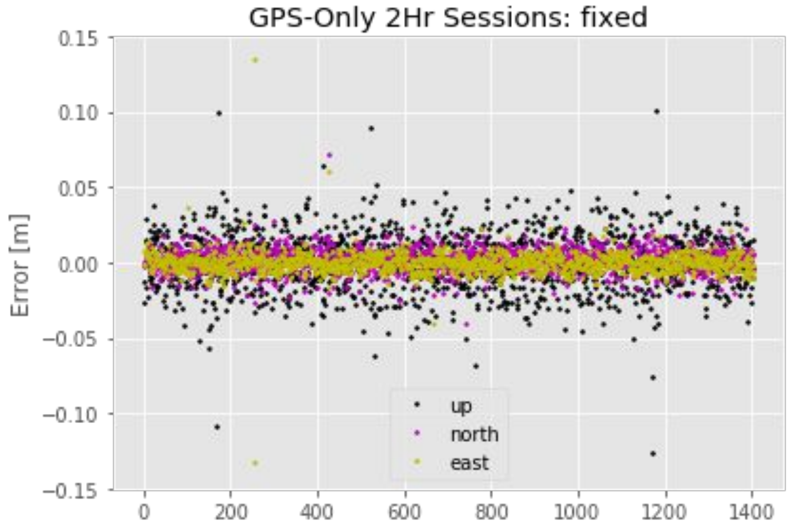

Sample Results- GPS+GAL

- ~30 baselines (< 200 km) selected from the NOAA CORS Network
- 45 x 2-Hr sessions (2021-001 - 2021-045)
- Single-baseline solutions evaluated against ITRF2014 station coordinate functions
- **Addition of Galileo improves results!**

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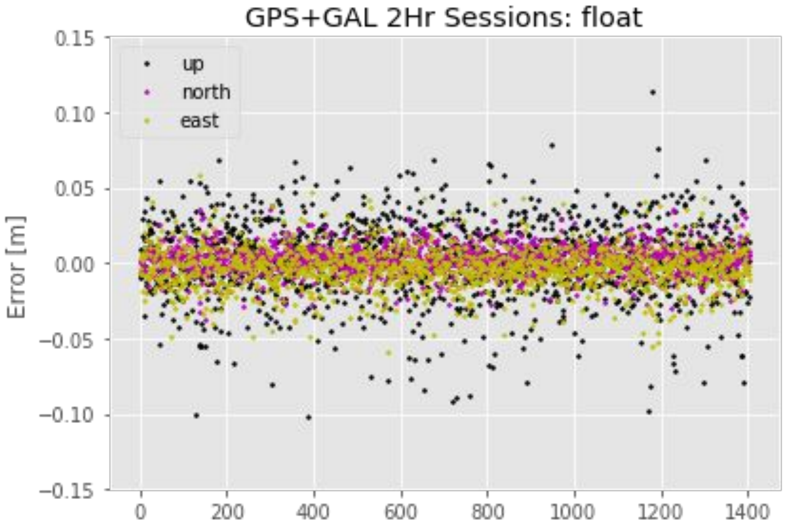

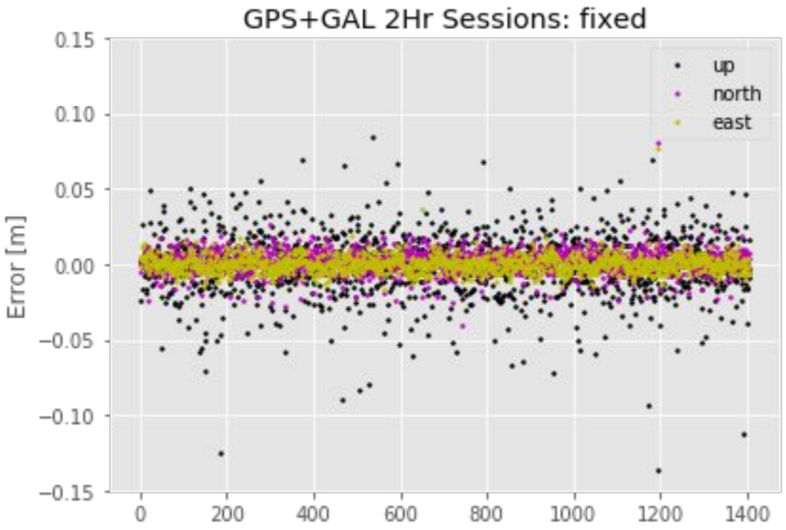

Next Steps

- Extend capabilities and testing beyond GPS & Galileo
- Testing w/ multi-baseline networks
- Improve ambiguity validation
- Integrate software into NGS services (e.g., OPUS)

References

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